

Inflation Target Uncertainty and Monetary Policy

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 - Historically inflation targets change
 - Uncertain optimal inflation rate

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 - Creates uncertainty around the future inflation target

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 - Response to perceived decline in r^*
 - Williams (2009), Blanchard et al. (2010), and Ball (2014)
 - Creates uncertainty around the future inflation target
- Why could this uncertainty matter?
 - Uncertainty in the future inflation target changes expected inflation
 - Individual, firm, and central bank decisions respond to such changes
 - Affects current outcomes regardless of the eventual resolution

Janet Yellen (June 2017)

"So it's that recognition that causes people to think we might be better off with a higher inflation objective. That is an important set, this is one of our most critical decisions and one we are attentive to evidence and outside thinking. It's one that we will be reconsidering at some future time... But I would say that this is one of the most important questions facing monetary policy around the world in the future."

This Paper

- Questions

- ① How does π^* uncertainty affect current inflation, output, and welfare?
- ② How should the central bank respond to inflation target uncertainty?
- ③ What happens if the central bank commits to changing the inflation target?

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- Approach

- Model inflation target uncertainty in a standard New Keynesian model
 - Policy rule with a regime specific inflation target
 - Exogenous Markov process determines regime
- Analytically solve the model without any additional uncertainty
- Numerically solve the stochastic model for full quantitative evaluation

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- Questions

- ① How does π^* uncertainty affect current inflation, output, and welfare?
 - A potential increase in π^* usually generates stagflationary dynamics
 - But may qualitatively differ depending on the current monetary policy rule
- ② How should the central bank respond to inflation target uncertainty?
 - A trade off in levels between inflation and output
 - Optimal policy adjusts the current inflation target
- ③ What happens if the central bank commits to changing the inflation target?
 - Anticipated change in π^* usually results in cyclical dynamics
 - Under an optimal time varying policy rule inflation monotonically adjusts

- Approach

- Model inflation target uncertainty in a standard New Keynesian model
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Literature

- Regime switches
 - Monetary policy uncertainty and optimal policy: Davig and Leeper (2007), Choi and Foerster (2016), Foerster (2016)
 - DSGE estimation: Schorfheide (2005), Liu, Waggoner, Zha (2011), Bianchi and Melosi (2016), Bianchi (2012a,b), and Davig and Doh (2014)
 - Solution methods and determinacy: Leeper and Zha (2003), Davig and Leeper (2007), Farmer, Waggoner, Zha (2009), Farmer, Waggoner, Zha (2011), and Foerster, Rubio-Ramirez, Waggoner, and Zha (2013)
- Uncertainty shocks
 - Bloom (2009), Baker et al. (2016), and Creal and Wu (2014), Ulrich (2012)

Model: Representative Household

- Preferences

$$u(C_t, N_t, \frac{M_t}{P_t}) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{\frac{M_t}{P_t}^{1-\nu}}{1-\nu}$$

- Composite consumption good

$$C_t = (\int_0^1 C_{it}^{\frac{e_t-1}{e_t}} di)^{\frac{e_t}{e_t-1}}$$

- Budget constraint

$$\int_0^1 P_{it} C_{it} di + M_t + \frac{1}{1+i_t} B_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t$$

Model: Firms

- Continuum of monopolistically competitive firms producing differentiated goods
- Technology

$$Y_{it} = A_t N_{it}^{1-\alpha}$$

- Calvo pricing: $1 - \omega$ adjust prices each period, others keep price constant
- Robustness: non-adjusters increase their previous price by the inflation target

Monetary Policy

- Interest rates are set according to a regime specific rule

$$i(s_t) = \phi_{\pi,s}\pi_t + \phi_{\pi',s}E_t\pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1)\pi_s^* + \phi_{x,s}x_t + \mu_t^I$$

- Regime is determined by a time invariant Markov process with transition matrix

$$\Pi = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1k} \\ p_{21} & p_{22} & \dots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \dots & p_{kk} \end{bmatrix} \quad k = 2, 3$$

- Central bank loss function

$$L_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)$$

$$EL = E(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)$$

Log-Linearized Model

- Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + (1 - \beta) \bar{\pi} + \mu_t^S, \quad (1)$$

where $\bar{\pi}$ is current regime's inflation target

- IS curve

$$x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + \mu_t^D \quad (2)$$

- Monetary policy

$$i(s_t) = \phi_{\pi,s} \pi_t + \phi_{\pi',s} E_t \pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi_s^* + \phi_{x,s} x_t + \mu_t^I \quad (3)$$

- Autoregressive shock processes

$$\mu_t^j = \rho_j \mu_{t-1}^j + \epsilon_t^j, \quad \epsilon_t^j \sim N(0, \sigma_j^2) \quad \forall j \quad (4)$$

- π is inflation, x is the output gap, i is the nominal interest rate

Solution Method

- Let Ω_t be the full information set and Ω_t^{-s} be the information set excluding the current regime, then

$$E_t \pi_{t+1} \equiv E[\pi_{t+1} | s_t = i, \Omega_t^{-s}] = \sum_{j=1}^k p_{ij} E[\pi_{j,t+1} | \Omega_t^{-s}]$$

$$E_t x_{t+1} \equiv E[x_{t+1} | s_t = i, \Omega_t^{-s}] = \sum_{j=1}^k p_{ij} E[x_{j,t+1} | \Omega_t^{-s}]$$

- The model in regime contingent notation:

$$x_{s,t} = \sum_{j=1}^k p_{sj} E_t x_{j,t+1} - \sigma^{-1}(i_{s,t} - \sum_{j=1}^k p_{sj} E_t \pi_{j,t+1}) + \mu_t^D$$

$$\pi_{s,t} = \beta \sum_{j=1}^k p_{sj} E_t \pi_{j,t+1} + \kappa x_{s,t} + (1 - \beta) \bar{\pi}_s + \mu_t^S$$

$$i_{s,t} = \phi_{\pi,s} \pi_t + \phi_{\pi',s} \sum_{j=1}^k p_{sj} E_t \pi_{j,t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi_s^* + \phi_{x,s} x_t + \mu_t^I$$

Quarterly Calibration

Parameter	Value	Parameter	Value
β	.99	ρ_I, ρ_S, ρ_D	.5
σ	2	σ_S	1.5
φ	1	σ_D	2
Ee_t	5	σ_I	2
α	.33	θ_x	.0408
ω	.66	θ_i	.25

Analytical Regime Switch Framework

- At $t = 0$, a surprise announcement that the central bank is considering permanently raising the inflation target to π^* with probability λ each period
 - Results are symmetric for a potential decrease in the inflation target
- If the inflation target is increased, it remains there forever
- Formally, the Markov process switches from $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to $\begin{bmatrix} 1 - \lambda & \lambda \\ 0 & 1 \end{bmatrix}$
- Assume monetary policy in regime two is determinate
- Only shock is the realization of the Markov process
 - Each regime is in a steady state
 - Let x_i denote the outcome in regime i

Regime Switch Outcomes

- Prior to the announcement, zero inflation steady state

$$x_t = \pi_t = 0$$

- After the announcement (regime 1): level shifts in inflation and output gap
 - Qualitatively depends on policy rule parameters
 - Quantitatively depends on π^* and transition probability
- After inflation target changes (regime 2): π^* inflation steady state

Key Forces

- Higher expected inflation
 - A potential increase in π^* increases expected inflation
 - Expected optimal price in future periods increases
 - Firms facing sticky prices raise prices immediately
- Different real interest rate
 - Higher expected inflation reduces real rates
 - Nominal rates adjust according to policy rule
 - Real rates rise if monetary policy responds enough to inflation
- A higher real interest rate reduces inflation and the output gap
 - Higher real interest rate incentivizes saving and reduces demand
 - Lower demand leads to lower consumption and output
 - Lower output reduces firm's marginal costs leading to lower prices

Regime 1 Response to Inflation Target Uncertainty

- Inflation
 - Rises by more than π^* if policy is passive
 - Rises by π^* if interest rates move one for one with inflation
 - Rises but by less than π^* if policy is active
 - Falls if $\phi_{\pi'}$ is large enough
- Output gap
 - Rises if policy is passive
 - Falls if policy is responsive enough to inflation
- Policy rule coefficients
 - A larger ϕ_{π} reduces the change in inflation and reduces output gap
 - A larger ϕ'_{π} reduces inflation and the output gap
 - A larger ϕ_x reduces inflation and the output gap if output gap is positive

Formal Version

Monetary policy response

- By changing π_1^* , any outcome along the the regime 1 Phillips Curve

$$\pi_1 = \frac{\pi^* \beta \lambda + \kappa x_1}{1 - \beta(1 - \lambda)}$$

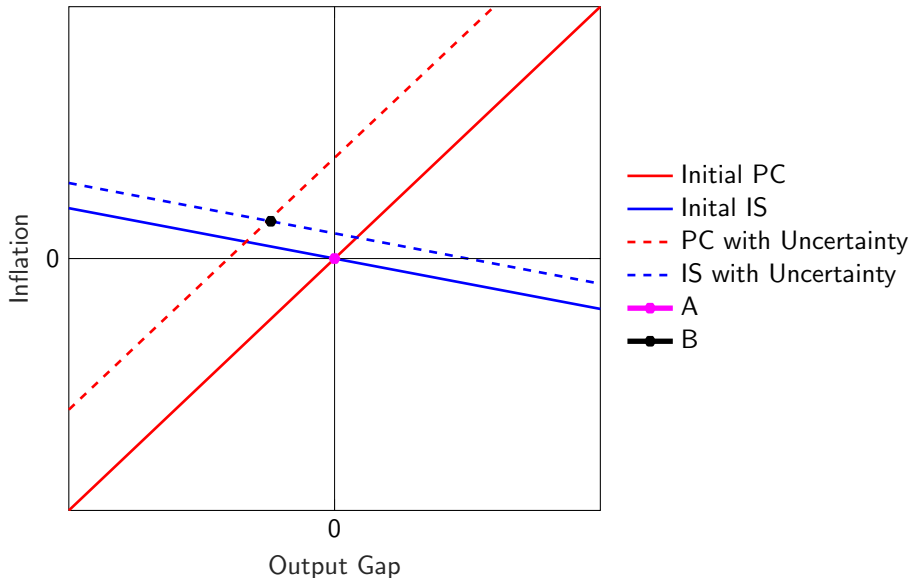
is achievable without affecting the volatility of inflation or output in a stochastic model

Stochastic Solution

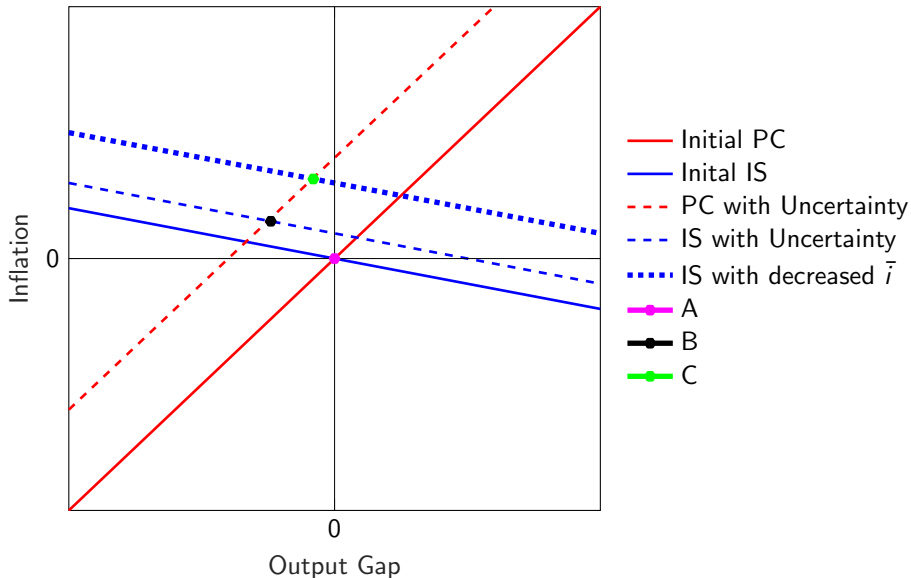
- A trade off in levels between inflation and output
- Reducing the constant in the policy rule (raising π_1^*) for the current regime will raise inflation and output in the current regime if monetary policy is active

Proposition 4

Potential Future Increase in the Inflation Target



Central Bank Responds by Raising the Inflation Target



Zero Inflation and Optimal Policy

- To achieve no change in inflation the central bank must generate a recession of magnitude $\frac{\beta\lambda\pi_2}{\kappa}$
- Optimal allocation is determined by loss function and can be anywhere on the Phillips Curve between
 - Zero inflation and a recession
 - Zero output gap and an increase in inflation
- Can be achieved by adjusting the current inflation target

Formulas

Stochastic Model

- Recall the central loss function

$$L_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)$$

- Single regime optimal policy rule

- For $\theta_x = .0408$ and $\theta_i = .25$, $i = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$

[Details](#)

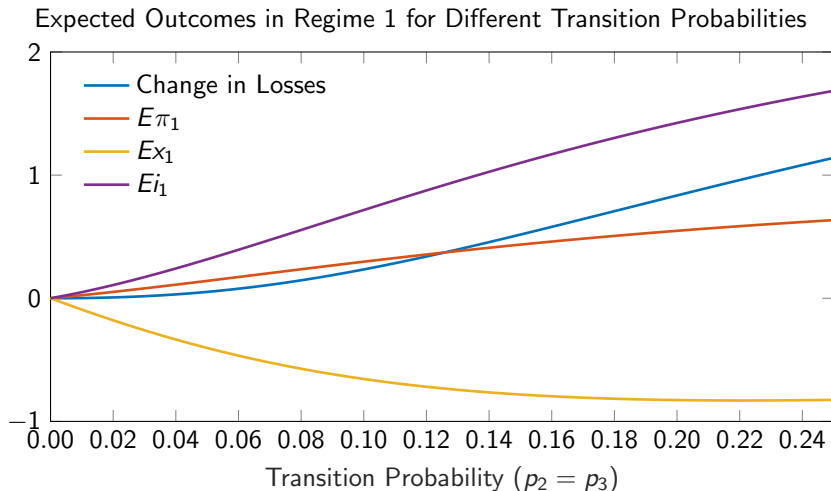
- Multiple regime optimal policy rule may differ

- The optimal policy rule is an approximation of the Ramsey optimal policy
- Regime switches create additional degrees of freedom and allows for a closer approximation of the optimal policy
- For welfare evaluations under the optimal response, comparisons are to the single inflation target policy rule that uses the same Markov structure

Quantitative Regime Switch Framework

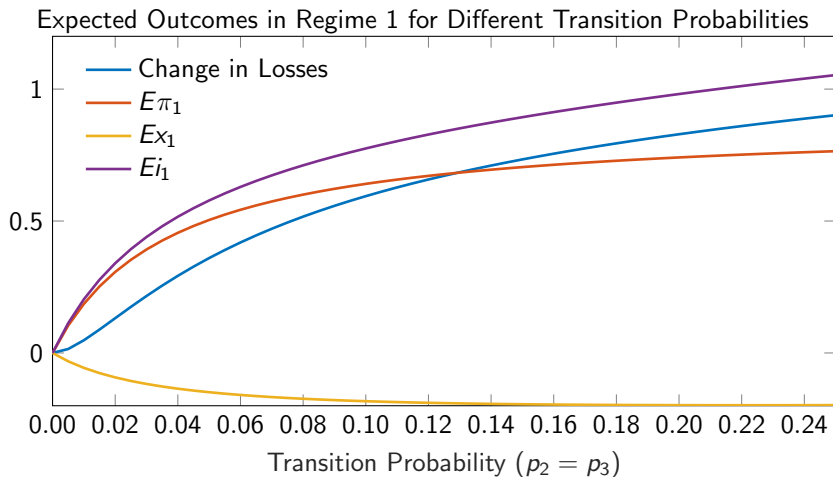
- Three regimes with ϕ_π , $\phi_{\pi'}$, and ϕ_x at single regime optimal values
- Initially in regime one, $\pi_1^* = 0$
- Probability p_2 to transition to regime 2 which has inflation target of 2%
- Probability p_3 to transition to regime 3 which has inflation target of 0%
- Regimes 2 and 3 are absorbing regimes
- For optimal policy considerations, comparison is to $\phi_{\pi,1}$, $\phi_{\pi',1}$, and $\phi_{x,1}$ that are optimal given a future regime shift to an absorbing regime
- Next
 - ① Holding monetary policy fixed, vary parameters
 - ② Allow central bank to respond

Main Specification



- Monetary policy causes an increase in real rates and a large recession
- More imminent resolution of the uncertainty magnifies the effects

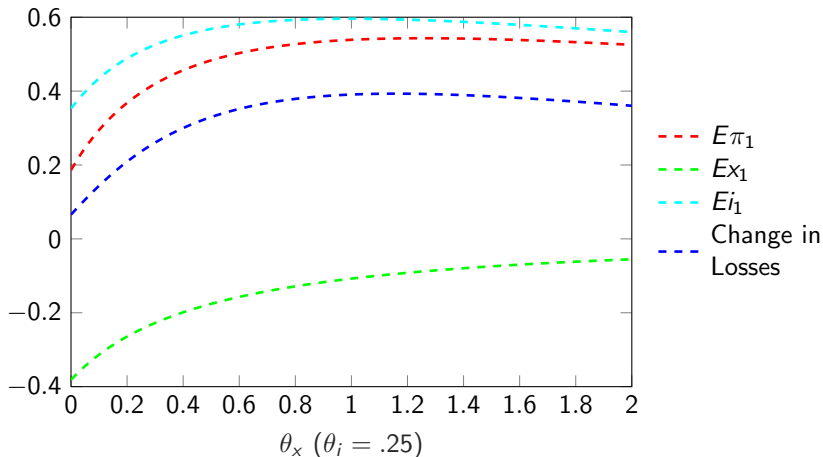
More Output Stabilizing Policy ($\theta_x = 1$)



- Real rates barely rise leading to a large rise in inflation
- Nominal rates actually increase more

Current vs Expected Inflation

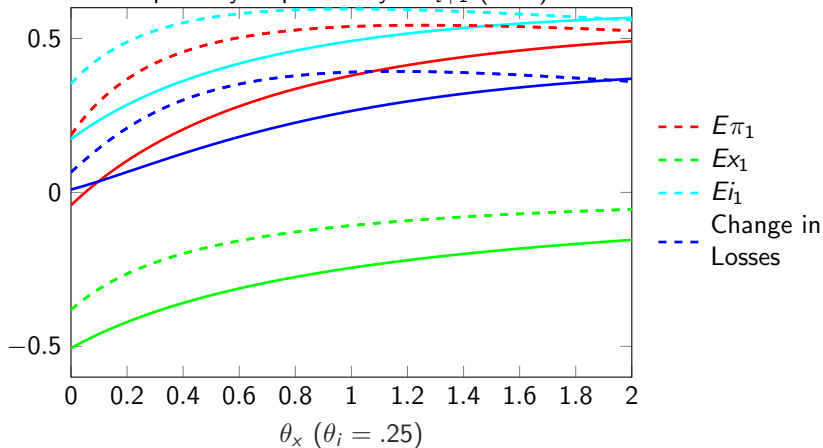
- Interest rates optimally respond only to π_t (dashed)



- A larger weight on output stability results in higher inflation

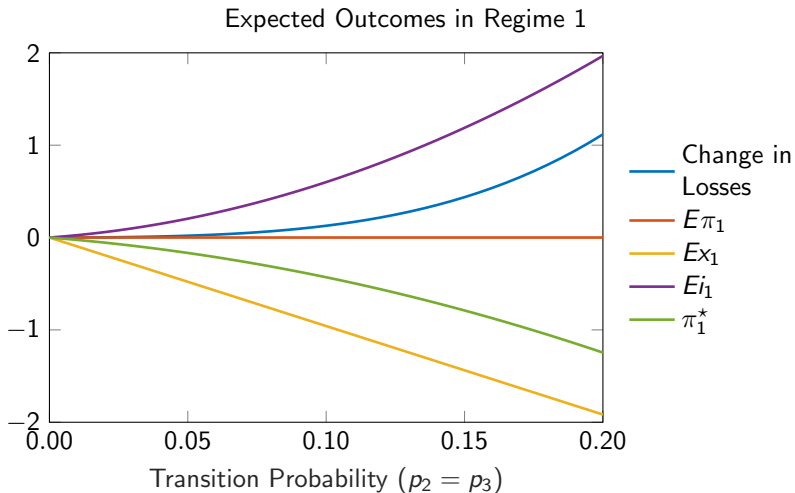
Current vs Expected Inflation

- Interest rates optimally respond only $E\pi_{t+1}$ (solid)



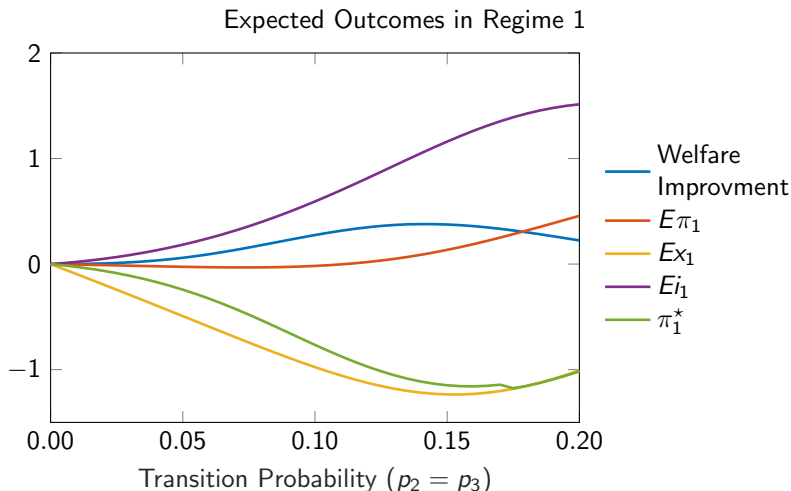
- Responding to $E\pi_{t+1}$ results in a larger recession and lower inflation

Central Bank Sets π_1^* s.t. $E\pi_1 = 0$



- Requires reducing π_1^* more than one for one with the transition probabilities

Optimal Monetary Policy Response



- The optimal response is a reduction in the current inflation target
- Results in higher real rates, a near zero inflation rate, and a large recession

Welfare Benefits of Optimal and Constrained Optimal Policy

Welfare Weights		% of Inflation Target Uncertainty Losses Eliminated			
θ_x	θ_i	Optimal Policy	Optimizing over		
			$\phi_\pi, \phi_{\pi'}, \phi_x$	π_1^*	π_1^* and Base Policy ¹
0.0000	0.25	81.12%	80.32%	81.12%	68.31%
0.0408	0.25	75.78%	72.49%	75.78%	63.13%
1.0000	0.25	62.18%	38.24%	62.18%	56.01%
4.0000	0.25	40.52%	17.42%	40.52%	36.37%
0.0408	0.05	11.82%	11.82%	11.82%	06.03%
0.0408	0.50	96.30%	83.99%	96.30%	90.95%

¹ Base policy: All other parameters in regime one are at their single regime optimums rather than the regime one optimal policy in the three regime model without uncertainty.

- Changing the inflation target is sufficient to minimize losses
- Optimal policy is most effective for large θ_i and small θ_x
- Effective of only changing other parameters is diminishing in θ_x
- With indexing similar optimal policy benefits, but require marginal adjustments of ϕ_π , $\phi_{\pi'}$, and ϕ_x

[Details](#)

Perfect Foresight of a Future Inflation Target Increase

- A potential resolution to the uncertainty is for the central bank to commit to changing the inflation target in the future
- At $t = 0$, a surprise announcement that at $t = T$ the inflation target will be permanently increased from 0 to π^*
- Deterministic equilibrium and $\phi_x = \phi_{\pi'} = 0$ (but ϕ_{π} arbitrary)
- What happens?
 - At $t = 0$ inflation and the output gap jump
 - For $t \geq T$, π^* steady state

Perfect Foresight Transition Path

Key forces:

- ① Higher $E\pi \rightarrow$ firms facing sticky prices raise prices
- ② Real rates fall, rise, or remain constant depending on $\phi_\pi \rightarrow$ change in m.c.
- ③ Anticipation: at T-2 agents fully anticipate T-1 outcomes

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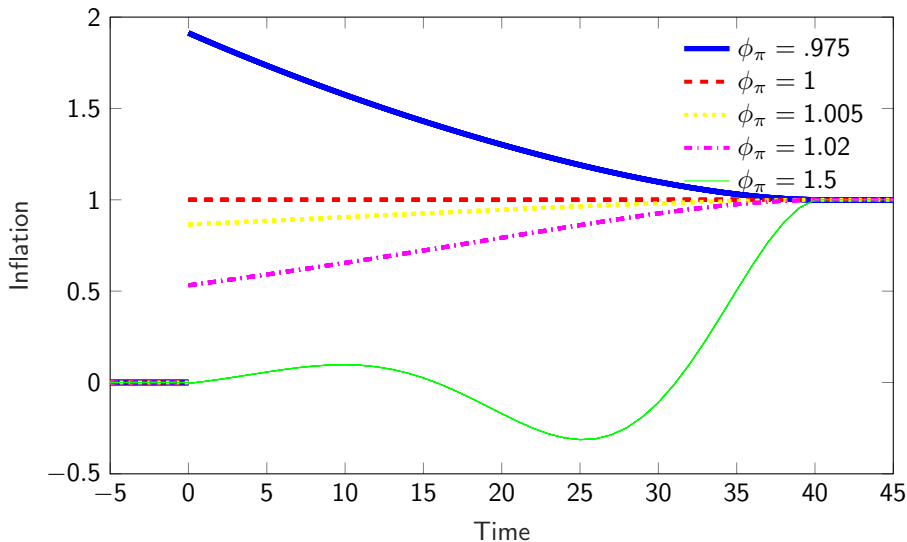
Outcomes:

- If $\phi_\pi < \bar{\phi}_\pi$, inflation and the output gap adjust monotonically
 - Multiple cases depending on whether ϕ_π is < 1 or > 1
 - Analogous to regime switch cases
- If $\phi_\pi > \bar{\phi}_\pi$, inflation and the output gap exhibit cyclical dynamics
 - Strong monetary policy response generates overshooting

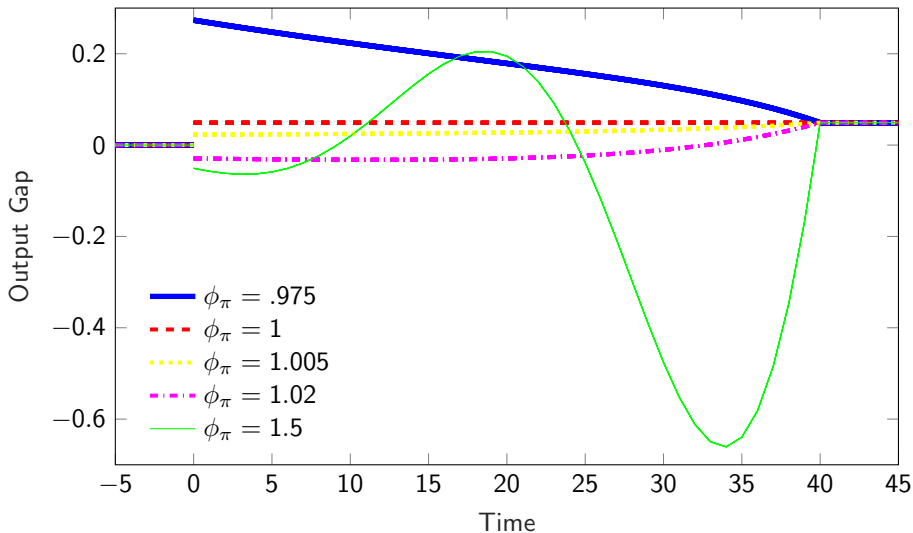
where $\bar{\phi}_\pi = \frac{1}{4\beta} \left(\frac{(1-\beta)^2}{\sigma-1\kappa} + 2(1-\beta) - \sigma^{-1}\kappa \right)$

Formal Version

Perfect Foresight Transition Path to $\pi_{40}^* = 1$: Inflation

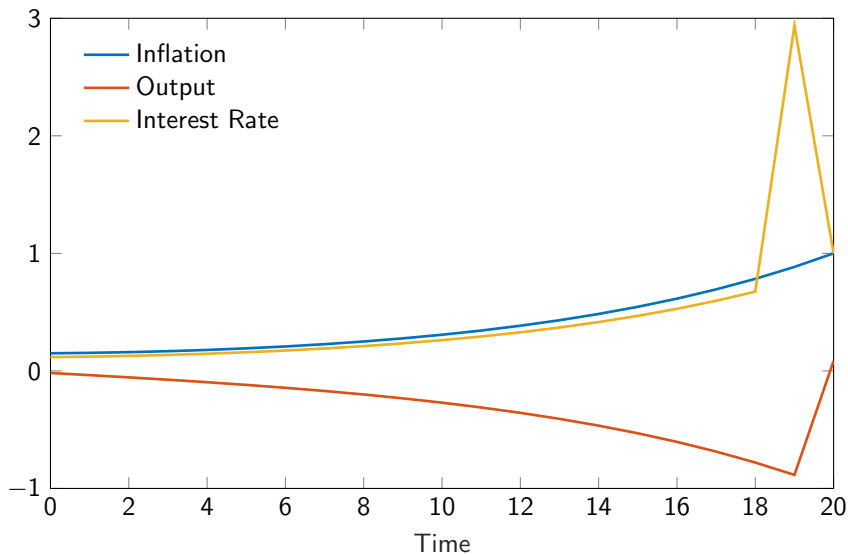


Perfect Foresight Transition Path $\pi_{40}^* = 1$: Output



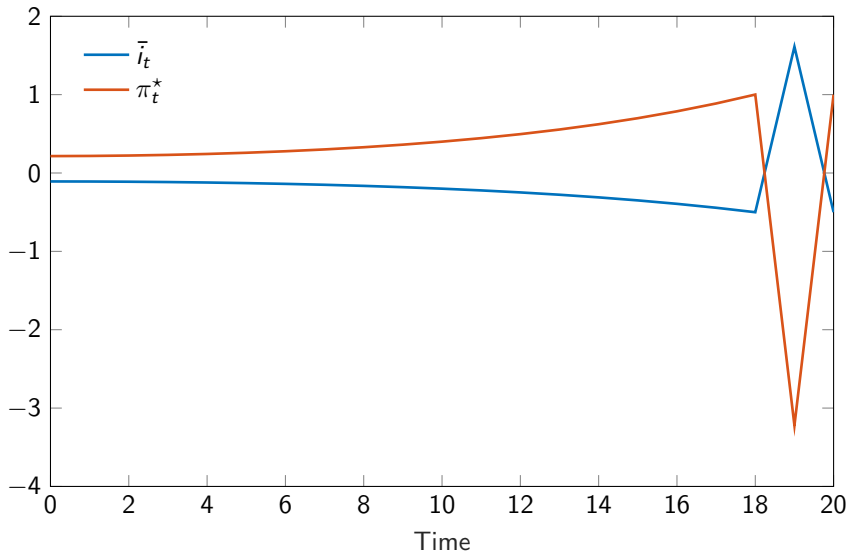
Perfect Foresight Transition Path with Optimal Policy

- Most active policy rules result in cyclical dynamics
 - $\bar{\phi}_\pi \approx 1.05$
 - Cyclical dynamics are suboptimal
- What is the optimal transition path?
 - Inflation monotonically adjusts to π^*
 - Output gap monotonically adjusts in the opposite direction

Optimal Transition Path ($\pi^* = 1$, $\theta_x = 1$ and $\theta_i = 0$)

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 - $\bar{\phi}_{\pi} \approx 1.05$
 - Cyclical dynamics are suboptimal
- What is the optimal transition path?
 - Inflation monotonically adjusts to π^*
 - Output gap monotonically adjusts in the opposite direction
- Implement by announcing a path for the inflation target along the entire transition path
 - Large non-monotonic change in interest rates the period before the new inflation target is achieved

Optimal Policy Along Transition Path ($\phi_\pi = 1.5$)

Conclusion

- A potential future increase in the inflation target generates upward pressure on inflation
- The monetary policy response generally raises real rates and leads to stagflation
- Creates a level trade-off between inflation and output with the outcome determined by the current inflation target
- Commitment to changing the inflation target at a certain point in time leads to cyclical fluctuations
 - Fluctuations eliminated under the optimal policy which relies on an announced time varying path for the inflation target

Regime 1: Active Policy

Define: $C_1 = 1 + \frac{1-\beta(1-\lambda)}{\kappa\sigma^{-1}} + \frac{\beta\phi_x}{\kappa}$ and $C_2 = 1 + \frac{(1-\beta)(1-\beta(1-\lambda))}{\kappa\sigma^{-1}}$

Monetary policy is active if $\phi_\pi + \phi_{\pi'} > 1$, then

- ① π_1 is increasing in π^* if $C_1 > \phi_{\pi'}$
 - $\phi_{\pi'}$ increases interest rates proportional to $(1-\lambda)\pi_1 + \lambda\pi_2$
 - Large $\phi_{\pi'}$ can raise i_1 enough for current inflation to fall
 - Large ϕ_π can't since if $\pi_1 < 0 \Rightarrow i_1 \downarrow$
- ② $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_{\pi'}} < 0$
- ③ $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_\pi} < 0$ if $C_1 > \phi_{\pi'}$
- ④ x_1 is decreasing in π^* if $C_2 < \beta\phi_\pi + \phi_{\pi'}$
 - Do interest rates rise enough to incentivize saving?
- ⑤ $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_x} > 0$ if $C_2 < \beta\phi_\pi + \phi_{\pi'} + \frac{\beta(1-\beta(1-\lambda)^2)}{\kappa}\phi_x$
- ⑥ If $\phi_\pi, \phi_{\pi'}, \phi_x$ are the same in both regimes, then $\pi_1 < \pi_2 \leq \pi^*$ and $x_1 < x_2$

Return

Regime 1: Passive Policy

Define

$$\bar{\phi}_{\pi} = 1 - \lambda \left(1 + \frac{1 - \beta(1 - \lambda)}{\sigma^{-1}\kappa} \right)$$

If monetary policy is passive ($\phi_{\pi} + \phi_{\pi'} \leq 1$) and $\phi_{\pi'} = \phi_x = 0$, then

- ① If $\phi_{\pi} = 1$, then $\pi_1 = \pi_2$ and $x_1 = \frac{1-\beta}{\kappa}\pi_2$
- ② If $\phi_{\pi} \in (\bar{\phi}_{\pi}, 1)$, then $\pi_1 > \pi_2$ and $x_1 > \frac{1-\beta}{\kappa}\pi_2$ and both approach infinity as ϕ_{π} is reduced to the lower bound
- ③ If $\phi_{\pi} < \bar{\phi}_{\pi}$, then stochastic equilibrium is indeterminate

Return

Proposition 4

Reducing the constant in the policy rule for the current regime, $\bar{i}_1$, will raise inflation and output in the current regime if

$$\phi_{\pi} + (1 - \lambda)\phi_{\pi'} + \phi_x \frac{1 - \beta(1 - \lambda)}{\kappa} > 1 - \lambda \left(1 + \frac{1 - \beta(1 - \lambda)}{\sigma^{-1}\kappa} \right) < 1$$

Return

Proposition 8

If

$$\frac{(1 - \beta)^2}{\sigma^{-1}\kappa} + \sigma^{-1}\kappa \geq 2(\beta\phi_\pi - 1) + 2\beta(\phi_\pi - 1),$$

then

$$\lambda = \frac{1 + \beta + \sigma^{-1}\kappa \pm \sqrt{(1 + \beta + \sigma^{-1}\kappa)^2 - 4\beta(1 + \sigma^{-1}\kappa\phi_\pi)}}{2(1 + \sigma^{-1}\kappa\phi_\pi)}$$

is a real number, and for $t \in [0, T]$

$$\pi_{T-t} = \pi^* \frac{\lambda_2^{t+1}(1 - \lambda_1) - \lambda_1^{t+1}(1 - \lambda_2))}{\lambda_2 - \lambda_1}$$

$$x_{T-t} = \pi^* \frac{\lambda_2^{t+1}(1 - \lambda_1)(\lambda_2 - \beta) - \lambda_1^{t+1}(1 - \lambda_2)(\lambda_1 - \beta)}{\kappa(\lambda_2 - \lambda_1)}$$

Otherwise λ is a complex number, and for $t \in [0, T]$

$$\pi_{T-t} = \frac{\pi^*(\phi_\pi - 1)\sigma^{-1}\kappa}{1 + \sigma^{-1}\kappa\phi_\pi} r^t \sum_{j=0}^{\infty} r^j \frac{\sin(\omega(t + 1 + j))}{\sin(\omega)},$$

where $r = \sqrt{\frac{\beta}{1 + \sigma^{-1}\kappa\phi_\pi}}$, $\omega = \cos^{-1}\left(\frac{1 + \beta + \sigma^{-1}\kappa}{2\sqrt{\beta(1 + \sigma^{-1}\kappa\phi_\pi)}}\right)$, and $x_{T-t} = \frac{\pi_{T-t} - \beta\pi_{T-t+1}}{\kappa}$

Proposition 9

If λ is real and $\pi^* > 0$, then

- ① If $\phi_\pi < 1$, then $\pi_{T-t} > \pi^*$, $x_{T-t} > \frac{1-\beta}{\kappa}\pi^*$, and both are monotonically decreasing.
- ② If $\phi_\pi = 1$, then $\pi_{T-t} = \pi^*$, $x_{T-t} = \frac{1-\beta}{\kappa}\pi^*$
- ③ If $\frac{1}{\beta} \geq \phi_\pi > 1$, then $0 < \pi_{T-t} < \pi^*$, $0 < x_{T-t} < \frac{1-\beta}{\kappa}\pi^*$, and both are monotonically increasing.
- ④ If $\phi_\pi > \frac{1}{\beta}$, then for small t ; $\pi_{T-t} < \pi^*$, $x_{T-t} < \frac{1-\beta}{\kappa}\pi^*$, and both are monotonically increasing. For large enough t , either or both π_{T-t} and x_{T-t} may be monotonically decreasing.

Return

Stochastic Solution

- Solving the full stochastic model it can be shown that the solutions for output and inflation are of the form

$$\pi_{j,t} = g_S e_{j,1} \mu_t^S + g_I e_{j,2} \mu_t^I + g_D e_{j,3} \mu_t^D + g_1 e_{j,4} a_j$$

$$x_{1,t} = f_{1,S} \mu_t^S + f_{2,I} \mu_t^I + f_{1,D} u_t^D + f_{1,1} a_1 + f_{1,2} a_2 - (1 - \beta) \kappa^{-1} \bar{\pi}_1$$

$$x_{2,t} = f_{2,S} \mu_t^S + f_{2,I} \mu_t^I + f_{2,D} u_t^D + f_{2,1} a_1 + f_{2,2} a_2 - (1 - \beta) \kappa^{-1} \bar{\pi}_2$$

- Then

$$\text{Var}(\pi_{j,t}) = g_S^2 e_{j,1}^2 \frac{\text{Var}(\epsilon_t^S)}{1 - \rho_S^2} + g_I^2 e_{j,2}^2 \frac{\text{Var}(\epsilon_t^I)}{1 - \rho_I^2} + g_D^2 e_{j,3}^2 \frac{\text{Var}(\epsilon_t^D)}{1 - \rho_D^2}$$

$$\text{Var}(x_{j,t}) = f_{j,S}^2 \frac{\text{Var}(\epsilon_t^S)}{1 - \rho_S^2} + f_{j,I}^2 \frac{\text{Var}(\epsilon_t^I)}{1 - \rho_I^2} + f_{j,D}^2 \frac{\text{Var}(\epsilon_t^D)}{1 - \rho_D^2}$$

- But the inflation targets only appear in the a_1 and a_2 terms.

Zero Inflation and Optimal Policy

- To achieve $\pi_1 = 0$, requires a recession of magnitude $\frac{\beta\lambda\pi_2}{\kappa}$ which can be accomplished by setting

$$\pi_1^* = \underbrace{\lambda\pi_2}_{\text{Impact of } \pi^* \text{ on } E_t\pi_{t+1}} \underbrace{\frac{1}{1 - \phi_\pi - \phi_{\pi'}}}_{\text{Direct effect}} \underbrace{\left(1 - \phi_{\pi'} + \frac{\beta\phi_x}{\kappa} + \frac{1 - \beta(1 - \lambda)}{\sigma^{-1}\kappa}\right)}_{\text{Equilibrium effect}}$$

- The optimal commitment policy in regime one will set the inflation target in regime one such that

$$x_1 = \frac{-\pi_2\lambda\beta}{\kappa + \kappa^{-1}(1 - \beta(1 - \lambda))^2\theta_x} \in \left[-\frac{\beta\lambda\pi_2}{\kappa}, 0\right]$$

$$\pi_1 = \frac{\pi_2\lambda\beta}{1 - \beta(1 - \lambda)} \frac{\kappa^{-1}(1 - \beta(1 - \lambda))^2\theta_x}{\kappa + \kappa^{-1}(1 - \beta(1 - \lambda))^2\theta_x} \in \left[0, \frac{\pi_2\lambda\beta}{1 - \beta(1 - \lambda)}\right]$$

Single Regime Optimal Policy

Table: Optimal Policy Rule Under a Single Regime

θ_x	θ_i	ϕ_π	$\phi_{\pi'}$	ϕ_x	$E\pi^2$	Ex^2	Ei^2	Losses
0.0408	0.2500	1.8255	1.1628	0.5057	5.9875	13.4033	25.7152	12.9632
0.0000	0.2500	1.8256	1.1628	0.2500	5.5686	17.0096	27.1140	12.3471
0.5000	0.2500	1.8255	1.1628	3.3829	8.6727	2.6147	22.8060	15.6816
1.0000	0.2500	1.8256	1.1628	6.5161	9.7410	1.0113	22.9697	16.4947
2.0000	0.2500	1.8254	1.1627	12.7805	10.5714	0.3279	23.3732	17.0705
0.0408	0.0500	9.9279	5.2140	1.5283	1.4561	31.6754	55.1211	5.5045
0.0408	0.5000	0.8128	0.6564	0.3778	9.9605	11.7571	14.6360	17.7582

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Monetary Policy Uncertainty

Table: Regime One and Two Outcomes with $i_1 = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$ and $p_{11} = p_{22} = .9$

i_2	$E\pi_1^2$	Ex_1^2	Ei_1^2	EL_1	$E\pi_2^2$	Ex_2^2	Ei_2^2	EL_2
$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	5.9875	13.4033	25.7152	12.9633	5.9875	13.4033	25.7152	12.9633
$2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	5.8953	13.4296	25.1804	12.7385	5.1295	15.4225	30.1017	13.2843
$1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	6.0968	13.3725	26.3523	13.2306	7.0804	11.6451	21.0922	12.8287
$1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$	5.9394	13.4170	25.4357	12.8459	5.5321	14.3956	27.9493	13.1069
$1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$	6.0396	13.3886	26.0181	13.0905	6.4977	12.4814	23.4373	12.8664
$1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t$	6.0491	13.3330	25.9876	13.0902	6.6329	9.0575	24.0142	13.0062
$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$	5.8944	13.5128	25.3164	12.7750	5.2176	21.8323	29.8090	13.5608

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Monetary Policy Uncertainty

Table: Expected Outcomes with $i_1 = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$ and $p_{11} = p_{22} = .9$

i_2	$E\pi^2$	Ex^2	Ei^2	Losses
$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	5.9875	13.4033	25.7152	12.9633
$2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	5.5124	14.4261	27.6411	13.0114
$1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	6.5886	12.5088	23.7222	13.0296
$1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$	5.7357	13.9063	26.6925	12.9764
$1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$	6.2686	12.9350	24.7277	12.9784
$1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t$	6.3410	11.1952	25.0009	13.0482
$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$	5.5560	17.6725	27.5627	13.1679

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Optimal Policy Response

Table: Optimal Regime 1 Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

i_1^*	i_2	$E\pi^2$	$E\pi^2$	Ei^2	Losses
$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t^*$	$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	5.9876	13.4029	25.7150	12.9633
$1.85927\pi_t + 1.1630E\pi_{t+1} + 0.5057x_t$	$2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	5.4778	14.4912	27.7677	13.0111
$1.8054\pi_t + 1.1439E\pi_{t+1} + 0.5057x_t$	$1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	6.6216	12.4503	23.5990	13.0294
$1.8454\pi_t + 1.1579E\pi_{t+1} + 0.5057x_t$	$1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$	5.7177	13.9397	26.7590	12.9763
$1.8183\pi_t + 1.1483E\pi_{t+1} + 0.5057x_t$	$1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$	6.2845	12.9061	24.6687	12.9784
$1.8298\pi_t + 1.1542E\pi_{t+1} + 0.5336x_t$	$1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t$	6.3637	11.0348	24.9350	13.0478
$1.8322\pi_t + 1.1495E\pi_{t+1} + 0.4680x_t$	$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$	5.5276	17.9038	27.6354	13.1672

There are alternative policy rules for regime one that result in the same losses in the two regime model.

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Optimal Policy Response

Table: Expected Per Period Regime 1 Loss Minimizing Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

i_1^*	i_2	$E\pi_1^2$	Ex_1^2	Ei_1^2	EL_1
$1.4114\pi_t + 0.9439E\pi_{t+1} + 0.4702x_t$	$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	7.0968	11.9718	20.9683	12.8275
$1.4160\pi_t + 0.9439E\pi_{t+1} + 0.4699x_t$	$2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	6.9723	12.0217	20.5802	12.6080
$1.4062\pi_t + 0.9438E\pi_{t+1} + 0.4706x_t$	$1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$	7.2444	11.9145	21.4314	13.0885
$1.4125\pi_t + 0.9432E\pi_{t+1} + 0.4700x_t$	$1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$	7.0351	11.9943	20.7491	12.7119
$1.4102\pi_t + 0.9445E\pi_{t+1} + 0.4705x_t$	$1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$	7.1634	11.9480	21.2063	12.9526
$1.4082\pi_t + 0.9444E\pi_{t+1} + 0.4778x_t$	$1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t$	7.1865	11.7591	21.1317	12.9494
$1.4164\pi_t + 0.9429E\pi_{t+1} + 0.4586x_t$	$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$	6.9620	12.3057	20.7275	12.6461

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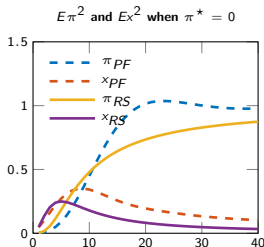
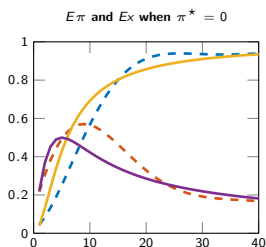
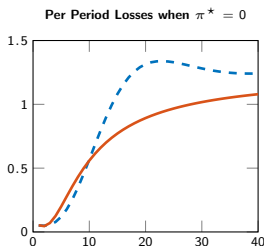
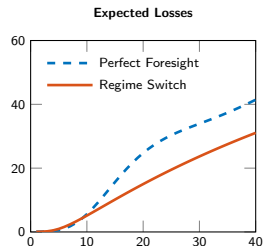
Welfare Benefits of Optimal and Constrained Optimal Policies: Indexing

Welfare Weights		% of Inflation Target Uncertainty Losses Eliminated			
θ_x	θ_i	Optimal Policy	Optimizing over		
			$\phi_\pi, \phi_{\pi'}, \phi_x$	π_1^*	π_1^* and Base Policy ¹
0.0000	0.2500	0.8202	0.8082	0.8165	0.6878
0.0408	0.2500	0.7733	0.7263	0.7645	0.6367
1.0000	0.2500	0.7699	0.3853	0.7218	0.6490
4.0000	0.2500	0.8478	0.1837	0.7383	0.6384
0.0408	0.0500	0.1287	0.1134	0.1198	0.0616
0.0408	0.5000	0.9962	0.8443	0.9726	0.9177

¹ Base policy: All other parameters in regime one are at their single regime optimums rather than the regime one optimal policy in the three regime model without uncertainty.

- Optimal response requires a change in both π_1^* and $\phi_\pi, \phi_{\pi'}, \phi_x$
- Only changing π_1^* captures most of the benefit of the optimal response

Anticipated Future 1% Decrease in the Inflation Target



$i_t = 1.5\pi_t - .5\pi_t^*$
Regime switch framework
results in:

- Stabilizing inflation at initial target increases losses
- Similar or lower expected total and per period losses
- Expected values are similar
- Variance is lower (none)

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